APPENDIX 6

1 - The evaluation of the complexity of the reception filters

Two cases have to be considered depending on whether it is sought or not to keep all the properties sought. Thus, for the *4n* + 3 and *4n* + 5 order filters:

- To keep null ISI, the matched pair, the phase linearity of the sending and reception filters, as shown in Figures 10, four lattices are used. These four lattices work at the lowest rate. Given the fact that for each lattice two multiplications and additions are necessary, except for each first cell (one multiplier and one adder), the operational complexity is (8n + 4) MPU and APU, plus a sign inversion.

- For a back-to-back system, simplifications are possible but only null ISI and the phase linearity of the sender filter are guaranteed. For the simplified *4n* + 3 and 4n + 5 order reception filters, the operational complexity corresponds then to that of a single lattice plus that of the three adders, the multiplier by g and the inverter. The computations performed at the lowest rate can therefore be estimated at (*2n* + 3) MPU, (*2n* + 5) APU and one inversion.

2 - A method of generic synthesis

As compared with the initially proposed method of synthesis, we now have a synthesis method that can be used, for all three types of solutions, to meet a frequency specification such as the one shown in Figure 2.

For the filter *F(z)* with a length *L*, different from *4n* + 1, in other words for an *N* order different from *4n* and for a fixed value of the fall-back factor ρ, we seek to minimize the cost function:

$$\Phi(F) = \sup\{w_P\left(1-|F(1)|\right)^2, w_C\left(\frac{\sqrt{2}}{2} - |F(e^{j\frac{\pi}{8}})|\right)^2, w_S \sup_{[\omega_S,\pi]} |F(e^{j\omega})|^2\}.$$

We use the FSQP (feasible sequential quadratic programming) algorithm developed by the team led by A.L. Tits [5] to search for the minimum of a set of non-linear even cost functions subjected to general, even and non-linear constraints. Since this method is a method of local optimization, the choice of an initial point is necessary. For a given weight *w* in the stopping band and a weight equal to 1 at the starting point and in the Nyquist frequency, namely $\pi/8$, we calculate an initial filter F^{init} that is optimal for the norm of the minimal value. This filter F^{init} is then used for direct computation of a set of lattice coefficients for a filter

producing a matched pair with null ISI. Since F^{init} does not itself produce a matched pair with null ISI, the identification between the two sets of coefficients, the transversal coefficients of F^{init} and the lattice coefficients is not exactly achieved. We therefore obtain a different filter, referenced F^{init}, whose lattice coefficients are considered to be the initial point of the optimization problem described by (63). For this problem, we fix $w_P = 1$, $w_C = 2$, $w_S = 0.5$.

Furthermore, for any lattice structure optimized for a given value ρ_0, we also use a continuation algorithm to obtain a lattice structure optimized for a value ρ_1, that is different in taking a sequence of intermediate values for ρ: the result of the lattice structure optimized for a value of ρ in the sequence is the initial point for the following value of ρ.

Figure 22 shows a higher attenuation in the stopping band than could be obtained for a fixed length and fall-back factor.

Let us consider for example the values $L = 43$ and $\rho = 0.5$: the best attenuation obtained by direct optimization is equal to 52.25 dB. The frequency response of the corresponding sending filter F^{opt} is given by Figure 23. The transversal coefficients and the coefficients of the lattice structure are given in Table 3.

Transversal coefficients

f_0	$-1.033698\ 10^{-4}$	f_{11}	$8.074275371845\ 10^{-3}$
f_1	$-3.6954272174\ 10^{-4}$	f_{12}	$1.5774357742194\ 10^{-2}$
f_2	$6.60549905248\ 10^{-4}$	f_{13}	$1.2603068512975\ 10^{-2}$
f_3	$-1.09979829792\ 10^{-4}$	f_{14}	$-6.358804372375\ 10^{-3}$
f_4	$4.31563793685\ 10^{-4}$	f_{15}	$-3.3366501346017\ 10^{-2}$
f_5	$6.08998879046\ 10^{-4}$	f_{16}	$-4.6538674170018\ 10^{-2}$
f_6	$6.39123817678\ 10^{-4}$	f_{17}	$-2.1044564354449\ 10^{-2}$
f_7	$-7.82626716253\ 10^{-4}$	f_{18}	$5.1214636348857\ 10^{-2}$
f_8	$-3.761395765875\ 10^{-3}$	f_{19}	$1.51186813816588\ 10^{-1}$
f_9	$-4.983816032565\ 10^{-3}$	f_{20}	$2.39358058463701\ 10^{-1}$
f_{10}	$-9.24070721312\ 10^{-4}$	f_{21}	$2.74720061562686\ 10^{-1}$

Lattice coefficients

α_1	-3.1848708937
α_2	$2.648518499\ 10^{-1}$
α_3	-2.9385797132
α_4	$-9.349457966\ 10^{-1}$
α_5	$9.246893837\ 10^{-1}$
α_6	$-1.308356726\ 10^{-1}$
α_7	3.6683584963
α_8	$-6.126257549\ 10^{-1}$
α_9	-1.2465410017
α_{10}	$-1.439677875\ 10^{-1}$
α_{11}	-3.5749582735

Table 3 : Lattice and transversal coefficients of the example with L = 43 and $\rho = 0.5$ ($f_{42-i} = f_i$)